

T-duality and HKT manifolds

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Abstract

We examine conformal rescaling and T-duality in the context of four-dimensional HKT geometries. The closure of the torsion forces the conformal factor to satisfy a modified harmonic equation. Because of this equation the conformal factors form non-commutative groups acting on the HKT geometries. Using conformal rescalings and T-duality transformations we generate from flat space new families of HKT geometries with tri-holomorphic Killing vectors. We also find ultraviolet-finite (4,0) supersymmetric sigma models which are not conformally invariant.

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1 Introduction

It is well known that supersymmetry and the target space geometry of sigma models are intimately related. In particular, Alvarez-Gaume and Freedman [1] showed some time ago that (4,4) supersymmetry in two dimensional sigma models restricts the target space geometry to be hyper-Kähler (HK). More recently an extension of the HK geometry, known as hyper-Kähler with torsion (HKT) [2], was found in (4,0) supersymmetric sigma models with Wess-Zumino coupling [3, 4, 5, 6, 7]. While the former geometry is defined with respect to the Levi-Civita connection, the latter is defined with respect to a connection with torsion, the torsion H being a closed three-form. In the limit of vanishing torsion one recovers from an HKT geometry an HK geometry.

Apart from their application to non-linear sigma models HKT geometries were recently found as moduli spaces for a certain class of black holes [8]. They also appear as the target spaces of a bound state of a D-string and D-five-branes in type I string theory [9]. Moreover, in [2] it was shown that there is a twistor space description associated to HKT geometries, resembling the ordinary twistor space for HK geometries [10].

The algebraic and differential constraints on the couplings of an HKT sigma model target manifold, the metric g and the locally defined two-form b , are much better understood in four dimensions than in higher dimensions, and a number of explicit examples of four-dimensional HKT geometries are known: In [11] it was shown, that there is a one-to-one correspondence between self-dual Einstein-Weyl spaces and HKT geometries. Moreover in [12] it was shown, that in the case of a four-dimensional HKT geometry with a tri-holomorphic Killing vector the associated three-space is a 'special' Einstein-Weyl space. The authors of [13] have used harmonic superspace methods to find generalizations of the Eguchi-Hanson and Taub-NUT metrics to include torsion. Furthermore, it was shown in [14] that there exists for every HK geometry with metric g an associated family of HKT geometries with metric $\tilde{g} = Ug$ and torsion $H = -*g dU$, provided that U is harmonic with respect to g . We generalize this observation, finding a family of conformally rescaled HKT geometries for every HKT geometry, where the conformal factor is in general *not* harmonic but rather satisfies a modified harmonic equation.

We focus our attention on HKT manifolds, admitting at least one tri-holomorphic Killing vector; furthermore we only consider four-dimensional HKT geometries. A tri-holomorphic Killing vector preserves all three complex structures. In addition, we always require an HKT geometry with a

tri-holomorphic Killing vector to have a torsion which is also invariant under the transformations generated by this Killing vector. Under these conditions one can T-dualize an HKT geometry along the Killing direction to find a new HKT geometry [15].

The torsion H can be expressed through a one-form u and a function u_0 . For $u_0 = 0$ the most general HKT geometry, which admits a tri-holomorphic Killing vector, is the conformal rescaling of the Gibbons-Hawking metric [16], whereas for $u = 0$ it is the geometry associated with monopoles on the three-sphere [17]. We generate both of these metrics from flat space by a series of consecutive T-duality transformations and conformal rescalings. Furthermore we find some new HKT geometries, which we give in equations (38), (43), and (46).

The organisation of the paper is as follows: In section 2, the HKT geometry is briefly reviewed, and we focus on the case of HKT geometries with a tri-holomorphic Killing vector. In section 3, we present the conformal rescaling of an HKT geometry and show that the conformal factors form non-commutative groups. In particular, we find the transformation rules for an HKT geometry with a tri-holomorphic Killing vector under conformal rescaling and T-duality. In section 4, starting from ordinary flat space we apply successively conformal rescaling and T-duality generating explicit examples of HKT manifolds. In section 5, we give some concluding remarks on the relation of conformal invariance versus finiteness in (4,0) supersymmetric sigma models.

2 The HKT geometry

A manifold M with a Riemannian metric g has an HKT structure if it fulfils the following requirements [2]:

(i) There is a triplet of complex structures $\{I_r; r = 1, 2, 3\}$, obeying the quaternionic algebra

$$I_r I_s = -\delta_{rs} + \epsilon_{rst} I_t . \quad (1)$$

(ii) The complex structures are covariantly constant

$$\nabla_\mu^{(+)}(I_r)_\nu{}^\rho = 0 \quad (2)$$

with respect to the connection with torsion

$$\Gamma^{(+)\mu}{}_{\nu\rho} = \Gamma^\mu{}_{\nu\rho} + \frac{1}{2} H^\mu{}_{\nu\rho} , \quad (3)$$

where Γ is the Levi-Civita connection with respect to the metric g .

(iii) The metric g is Hermitian with respect to all complex structures

$$g_{\rho(\mu}(I_r)_{\nu)}{}^{\rho} = 0 . \quad (4)$$

In the classical theory the torsion is a closed three-form, whereas in the quantum theory and in particular in the context of the anomaly cancellation mechanism, the (classical) torsion receives corrections [18, 19] and the new torsion is not a closed three-form anymore. Throughout this paper we take the torsion to be closed, which allows the local definition of a two-form b as

$$H_{\mu\nu\rho} = 3 \partial_{[\mu} b_{\nu\rho]} . \quad (5)$$

In particular in four dimensions the torsion three-form H is dual to a one-form v

$$H_{\mu\nu\rho} = \epsilon_{\mu\nu\rho}{}^{\lambda} v_{\lambda} , \quad (6)$$

where $\epsilon_{\mu\nu\rho\lambda}$ is the totally anti-symmetric tensor with respect to the metric g . We remark that the conditions (i)-(iii) are precisely the conditions necessary for the invariance of the action of a sigma model under (4,0) supersymmetry and the closure of the (4,0) supersymmetry algebra.

Now we assume that the sigma model is invariant under some target space transformations generated by a vector field X . Equivalently X is an Killing vector field, which leaves the torsion H and the complex structures I_r invariant

$$\mathcal{L}_X I_r = 0, \quad \mathcal{L}_X H = 0 . \quad (7)$$

We call Killing vectors, which satisfy these equations, tri-holomorphic. Next, we choose coordinates $\{x_0, x_i; i = 1, 2, 3\}$ on the target space M adapted to the Killing vector field X , i.e.

$$X = \frac{\partial}{\partial x_0} . \quad (8)$$

The metric on M in this adopted coordinate system can be written as

$$ds^2 = V^{-1}(dt + w_i dx^i)^2 + V \gamma_{ij} dx^i dx^j , \quad (9)$$

where γ is the three-dimensional metric on the manifold of Killing vector trajectories, M_3 , V is a function and w is a one-form on M_3 . Following the notations of [12] we write the torsion H in terms of a function u_0 and a one-form u on M as

$$\begin{aligned}
H_{0ij} &= \epsilon_{ij}{}^k u_k \\
H_{ijk} &= \epsilon_{ijk}(-V u_0 + w \cdot u) .
\end{aligned} \tag{10}$$

We choose an orientation such that $\epsilon_{0123} = +\sqrt{g} = +V\epsilon_{123} = +V\sqrt{\gamma}$. The closure of the torsion H implies that the divergence of the one-form u vanishes

$$\nabla^i u_i = 0 , \tag{11}$$

where ∇_i is the Levi-Cevita connection with respect to the three-metric γ .

Hence an HKT geometry admitting a tri-holomorphic Killing vector is completely specified by a three-metric γ , and the tensors V, w, u_0 and u on M_3 , which fulfil the following restrictions [12]:

(i) V and w satisfy a monopole-like equation

$$2\partial_{[i} w_{j]} = \epsilon_{ij}{}^k (\partial_k V - V u_k) . \tag{12}$$

(ii) (γ_{ij}, u_i) define a three-dimensional Einstein-Weyl geometry with a Ricci tensor

$$R_{ij} - \nabla_{(i} u_{j)} + u_i u_j = \gamma_{ij} \left(\frac{1}{2} u_0^2 + |u|^2 \right) . \tag{13}$$

(iii) (u_0, u_i) satisfy a monopole-like equation analogous to the one for (V, w)

$$2\partial_{[i} u_{j]} = \epsilon_{ij}{}^k (u_0 u_k - \partial_k u_0) . \tag{14}$$

An Einstein-Weyl space with the one-form u restricted by (11) and (14) and the Ricci tensor given by (13) is called a *special* Einstein-Weyl space.

By taking the divergence of (12) one finds that V is a solution of the modified harmonic equation

$$\nabla^i \nabla_i V = u^i \nabla_i V, \tag{15}$$

which reduces to the ordinary harmonic equation when $u = 0$. A similar equation follows from (14) for u_0 as

$$\nabla^i \nabla_i u_0 = u^i \nabla_i u_0 . \tag{16}$$

3 Conformal rescaling and T-duality

We start with the discussion of the conformal rescaling of a general four-dimensional HKT geometry. Let (g^1, H^1) be an HKT geometry, then there is an HKT geometry (g^2, H^2) given by

$$g^2 = U g^1 \quad (17)$$

$$H^2 = -^{*1}dU + U H^1, \quad (18)$$

provided that the non-zero conformal factor U satisfies

$$\square_1 U = (g^1)^{\mu\nu} (v_1)_\mu \partial_\nu U, \quad (19)$$

where the Laplace operator \square_1 is taken with respect to the metric g_1 . Note the factor of U in front of H^1 in (18). This factor arises because one of the indices of H is lowered in the first and second geometry with different metrics to find the torsion as a three-form. This factor is also responsible for the fact that the conformal factor U satisfies in general the modified harmonic equation (19) rather than a harmonic equation as it was claimed in [12].

The modified harmonic equation for U is of such a form that the conformal factors, to be precise the pairs $(U^\alpha, H^\alpha; \alpha = 1, 2, 3, \dots)$, form a group that act on the space of the conformal families of HKT geometries. In fact, the operation of conformal rescaling closes separately on each conformal family of HKT geometries. The group properties for the metrics g^α can be easily checked to be

$$\begin{aligned} g^3 &= U^2 g^2 = U^2 (U^1 g^1) = (U^2 U^1) g^1 && \text{closedness} \\ g^4 &= (U^3 U^2) U^1 g^1 = U^3 (U^2 U^1) g^1 && \text{associativity} \\ g^2 &= U^1 g^1 \Leftrightarrow \exists U^0 = 1 \text{ s.t. } g^2 = (U^0 U^1) g^1 && \text{existence of unity} \\ g^2 &= U^1 g^1 \Leftrightarrow \exists (U^1)^{-1} \text{ s.t. } g^1 = (U^1)^{-1} g^2 && \text{existence of inverse,} \end{aligned}$$

where the torsions H^α are related to the metrics g^α as in (18). The group properties for the torsion and the conformal factor are similar to the ones for the metric factors; for example for the closure of the group one can show:
(i) The torsion H^3 defined by

$$H^3 = -^{*2}dU^2 + U^2 H^2, \quad (20)$$

can be rewritten as

$$H^3 = -{}^*d(U^2U^1) + (U^2U^1)H^1, \quad (21)$$

provided that the torsion H^2 is given as in (18). (ii) The conformal factor (U^2U^1) is a harmonic function with respect to the metric g^1 if the conformal factors U^1 and U^2 are harmonic functions with respect to the metrics g^1 and g^2 , respectively. Similarly one can proof the remaining group properties for the torsion and the conformal factor. Note, that the group is not commutative because in

$$g^3 = U^2(U^1g^1) = U^1(U^2g^1) \quad (22)$$

$g = U^2g^1$ is in general not an HKT metric.

Let us assume that the HKT geometry admits a tri-holomorphic Killing vector. We examine in detail, how such a geometry transforms under conformal rescaling and T-duality. Let us first consider conformal rescaling of a tri-holomorphic HKT geometry.

Starting with an HKT geometry $((\gamma^1)_{ij}, V^1, (w^1)_i, (u^1)_0, (u^1)_i)$ we rescale the metric g^1 by a conformal factor U to find

$$(\gamma^2)_{ij} = U^2(\gamma^1)_{ij}, \quad V^2 = U^{-1}V^1, \quad (w^2)_i = (w^1)_i. \quad (23)$$

For the second geometry to be again tri-holomorphic, U is restricted $U = U(x^i)$ and the dual torsion follows as

$$(u^2)_i = (u^1)_i - \partial_i(\ln U), \quad (u^2)_0 = U^{-1}(u^1)_0. \quad (24)$$

Equation (19) reduces to the three-dimensional modified harmonic equation

$$\nabla^i \nabla_i U = u^i \nabla_i U. \quad (25)$$

Note, that for $u_0 = 0$ equation (14) implies that u_i is closed, $\partial_{[i}u_{j]} = 0$. Thus one can find locally a function f on M such that $u_i = \partial_i f$. Then it follows from (24) that this geometry is conformally related to an HK geometry. Since the most general HK geometry with a tri-holomorphic Killing vector is the Gibbons-Hawking metric, an HKT geometry with $u_0 = 0$ is necessarily the conformal rescaling of the Gibbons-Hawking metric.

The modified harmonic equation for U (25) is similar to the equations (15), and (16) for V , and U_0 , respectively. To explain the fact that U and V satisfy similar equations we remark that in every conformal family of HKT geometries there are those for which V is a constant. ($V = \text{const}$ is a special

solution of (15).) To find all the other metrics in the same conformal family with non-constant V we can conformally rescale the constant ones, i.e. we have to solve the modified harmonic equation for U (25). Thus solving (25) for U is equivalent to solving in general (15) for V . The fact that V and u_0 satisfy similar equations enables one to find for every three-dimensional special Einstein-Weyl space an associated 'minimal' four-metric

$$V = -\frac{u_0}{c}, \quad w = \frac{u}{c} \quad (26)$$

for any constant c . In fact, every minimal HKT geometry is conformally related to an HK geometry [12].

We now examine the transformation of HKT manifolds, which admit a tri-holomorphic Killing vector, under T-duality. Let us start again with an initial HKT geometry admitting a tri-holomorphic Killing vector. Then T-dualizing this geometry leads to [15]

$$(\gamma^3)_{ij} = (V^1)^2 (\gamma^1)_{ij}, \quad V^3 = (V^1)^{-1}, \quad (w^3)_i = (b^1)_{0i}, \quad (27)$$

where $(b^1)_{0i}$ is a solution of the equation

$$2\partial_{[i}(b^1)_{j]0} = (\epsilon^1)_{ij}{}^k (u^1)_k. \quad (28)$$

There always exists such a $(b^1)_{0i}$ because the integrability condition for (28) is just (11). The new torsion follows as

$$(u^3)_i = (u^1)_1 - \partial_i(\ln V^1), \quad (u^3)_0 = V^{-1}(u^1)_0. \quad (29)$$

One can explicitly show that the conformal rescaling (23) and T-dualization (27) of an initial geometry give rise to further HKT geometries, iff the initial geometry itself is HKT, i.e. equations (11) - (14) are invariant under conformal rescaling and T-duality, as expected on general grounds.

4 HKT metrics from flat space

We construct explicit examples of HKT manifolds with a tri-holomorphic Killing vector using a string of consecutive T-duality transformations and conformal rescalings. Let us start with flat space as a trivial example of an HKT geometry ¹

¹ We can equally well take the space $S^1 \times \mathbb{R}^3$ as a starting point and the periodic identification of the Killing direction goes through all T-dualizations and conformal rescalings.

$$\begin{aligned} g^1 &= (dx_0)^2 + dx \cdot dx \\ v^1 &= 0 . \end{aligned} \tag{30}$$

Now conformally rescale the metric (30) as in (23) with $U^1 = U^1(x^i)$ satisfying $\nabla^i \nabla_i U = 0$ to find

$$\begin{aligned} g^2 &= U^1 \left((dx_0)^2 + dx \cdot dx \right) \\ (u^2)_0 &= 0 , (u^2)_i = -\partial_i (\ln U^1) . \end{aligned} \tag{31}$$

A one-form w^1 on M_3 , defined as

$$2\partial_{[i}(w^1)_{j]} = (\epsilon^1)_{ij}{}^k \partial_k U^1 , \tag{32}$$

always exists because the integrability condition for (32) is that U is harmonic. In a next step we T-dualize the geometry (31) to find

$$\begin{aligned} g^3 &= (U^1)^{-1} (dx_0 + (w^1)_i dx^i)^2 + U^1 dx \cdot dx \\ v^3 &= 0 , \end{aligned} \tag{33}$$

This is the well known HK metric of Gibbons and Hawking [16]. Let us write the flat metric on M_3 in spherical polar coordinates

$$\gamma^3 = dr^2 + r^2 (d\phi^2 + \sin^2 \phi d\psi^2) \tag{34}$$

and take U^1 as the one-centre Greens function on M_3

$$U^1 = c_0 + \frac{c_1}{r} \tag{35}$$

with c_0 and c_1 as non-negative constants. Then it can be arranged that

$$w^1 = -c_1 \cos \phi d\psi . \tag{36}$$

The Gibbons-Hawking metric (33) can be further conformally rescaled by U^2 , which satisfies a similar harmonic equation on the flat three-space as U^1 . In particular, we can take U^2 and w^2 to be of the same form as U^1 and w^1

$$\begin{aligned}
U^2 &= c_2 + \frac{c_3}{r} \\
w^2 &= -c_3 \cos \phi d\psi
\end{aligned} \tag{37}$$

with c_2 and c_3 as non-negative constants. Changing variables from r to $z = \ln r$, $-\infty < z < +\infty$, the conformal rescaling leads to

$$\begin{aligned}
g^4 &= \frac{c_3 + c_2 e^z}{c_1 + c_0 e^z} (dx_0 - c_1 \cos \phi d\psi)^2 + \\
&\quad \frac{c_1 + c_0 e^z}{c_3 + c_2 e^z} [(c_3 + c_2 e^z)^2 (dz^2 + d\phi^2 + \sin^2 \phi d\psi^2)] \\
(u^4)_0 &= (u^4)_\phi = (u^4)_\psi = 0, \quad (u^4)_z = \frac{c_3}{c_3 + c_2 e^z} .
\end{aligned} \tag{38}$$

In the generic case of all four constants $\{c_1, c_2, c_3, c_4\}$ to be positive, the metric factor V^4 is regular but the three-metric is singular at $z = +\infty$, although the singularity is in an infinite proper distance. For $c_0 = c_2 = 0$, $c_1 = \lambda$ and $c_3 = 1$ the metric and the dual torsion (38) become

$$\begin{aligned}
g^5 &= \lambda^{-1} (dx_0 - \lambda \cos \phi d\psi)^2 + \lambda (dz^2 + d\phi^2 + \sin^2 \phi d\psi^2) \\
(u^5)_0 &= (u^5)_\phi = (u^5)_\psi = 0, \quad (u^5)_z = 1 .
\end{aligned} \tag{39}$$

This geometry has a constant $V^5 = \lambda$ and its three-geometry is $\mathbb{R} \times S^2$. Though v^5 is constant, H^5 is not because ϵ_{ijk} is not constant on $\mathbb{R} \times S^2$. The coordinate z can be periodically identified to give the compact version of the three-space, $S^1 \times S^2$. Both three-geometries as well as their associated four-geometries are *non-singular*. The effect of a further T-dualization of the geometry (38) is just to interchange the constants (c_0, c_1) with (c_2, c_3) . Therefore the geometry (39) with $\lambda = 1$ is T-self-dual. Thus we cannot generate from (38) any other HKT manifolds by further conformal rescalings or T-dualizations, i.e we are confined to one particular conformal family.

Let us start again with the flat four-space (30) but rewritten as

$$\begin{aligned}
g^6 &= dr^2 + r^2 d\tilde{\Omega}_3^2 \\
v^6 &= 0 ,
\end{aligned} \tag{40}$$

where $d\tilde{\Omega}_3^2$ is the volume element on the unit three sphere, S^3 . We perform a first conformal rescaling of (40) by a harmonic function U^3 which depends on r only,

$$U^3 = \frac{R^2}{r^2} \quad (41)$$

with R a poitive constant. Thus the rescaled geometry is

$$\begin{aligned} g^7 &= dx_0^2 + d\Omega_3^2 \\ (u^7)_0 &= \frac{2}{R}, \quad (u^7)_i = 0, \end{aligned} \quad (42)$$

where $x_0 = R \ln r$, and $d\Omega_3^2$ is the volume element on the three-sphere with radius R , $d\Omega_3^2 = R^2 d\tilde{\Omega}_3^2$. Note that this conformal rescaling is not of the form $U = U(x^i)$ as all the others are. With the former type of conformal rescaling we have generated a non-zero u_0 , whereas with the latter we cannot make u_0 non-zero. In fact, the geometry (42) is a member of a new conformal family. Even though the conformal rescaling does not preserve a tri-holomorphic Killing vector, after the coordinate change from r to x_0 , the geometry is cast in the form of an HKT geometry with a tri-holomorphic Killing vector. Further rescaling the metric (42) by U^4 gives

$$\begin{aligned} g^8 &= U^4(dx_0^2 + d\Omega_3^2) \\ (u^8)_0 &= \frac{2}{U^4 R}, \quad (u^8)_i = -\partial(\ln U^4), \end{aligned} \quad (43)$$

where U^4 is chosen to be a harmonic function on the three-sphere. Define w^4 by

$$2\partial_{[i}(w^4)_{j]} = (\epsilon^4)_{ij}{}^k \partial_k U^4 \quad (44)$$

with ϵ^4 taken with respect to $d\Omega_3^2$. T-dualization of (43) leads to

$$\begin{aligned} g^9 &= (U^4)^{-1}(dx_0 + (w^4)_i dx^i)^2 + U^4 d\Omega_3^2 \\ (u^9)_0 &= \frac{2}{R}, \quad (u^9)_i = 0. \end{aligned} \quad (45)$$

This is the unique HKT geometry with $u = 0$. This geometry was given in [17] with $R = \lambda^{-1}$. The three-geometry of (45) is that of a three-sphere of constant radius R . The global behaviour of (45) was studied in [17] for the case of a geometry, which is $u(2)$ -invariant. It was found that all geometries of this kind are singular except the WZW model with target space

$SU(2) \times U(1)$, which has a constant V and a vanishing w . It seems likely that all the remaining metrics of the type (45) are also singular. Further rescaling by a conformal factor U^5 , which is again a harmonic function on S^3 , results in

$$\begin{aligned} g^{10} &= \frac{U^5}{U^4} \left(dx_0 + (w^4)_i dx^i \right)^2 + \frac{U^4}{U^5} \left[(U^5)^2 d\Omega_3^2 \right] \\ (u^{10})_0 &= \frac{2}{U^5 R}, \quad (u^{10})_i = -\partial_i (\ln U^5) . \end{aligned} \quad (46)$$

We have checked that the singularities of the metric (45), which is also u_2 -invariant, cannot be resolved by such a conformal rescaling. Note, that the effect of T-duality on (46) is to interchange the pairs (U^4, w^4) with (U^5, w^5) which resembles the behaviour of (39) under T-duality. Thus again applying our techniques to (46) does not yield any new geometries.

5 Concluding remarks

It has been shown in [7, 20] by a power counting argument that (4,0) supersymmetric sigma models are ultraviolet finite. The condition for one-loop ultraviolet finiteness is [21, 22]

$$R_{ij}^{(+)} = \nabla_{(i} V_{j)} + \frac{1}{2} T_{ij}{}^k V_k + \partial_{[i} \lambda_{j]} , \quad (47)$$

where $R^{(+)}$ is the curvature associated with $\Gamma^{(+)}$, and V_i and λ_i are one-forms on M . In comparison, the condition for one-loop conformal invariance is [22]

$$R_{ij}^{(+)} = \nabla_i \nabla_j \phi + \frac{1}{2} T_{ij}{}^k \nabla_k \phi \quad (48)$$

for some function ϕ on M . The two conditions coincide only if V_i is a closed one-form, i.e. one-loop conformal invariance implies one-loop ultraviolet finiteness but not vice-versa.

The Ricci-tensor for an HKT geometry is given in [13] as

$$R_{ij}^{(+)} = \nabla_j^{(+)} v_i , \quad (49)$$

where v_i is the dual torsion (6). It follows that the condition for one-loop ultraviolet finiteness (47) is fulfilled for any HKT geometry with $\lambda_i = -v_i$ and $V_i = v_i$, whereas the condition for one-loop conformal invariance (48)

is fulfilled only for geometries for which one can set $\nabla_i \phi = v_i$. Thus all HKT geometries are one-loop ultraviolet finite, but only conformally rescaled HK geometries are one-loop conformally invariant. Moreover in [23] it was shown that for compact target spaces any ultraviolet finite sigma model is conformally invariant. We remark though that the (4,0) supersymmetric sigma model with target space the HKT geometry (45) is ultraviolet finite but not conformally invariant.

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